

Lab-3

STIFFNESS AND NATURAL FREQUENCY CALCULATIONS

ME159-ENGINEERING LAB

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By: Abdullah Alhajri

Group members:

Mandeep

Amanda

Abdulrahman

Instructor:

Sreekanth Rudraraju

California State University, Fresno
Department of Mechanical Engineering

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INTRODUCTION:

Any physical system consists of many elements like springs, beams (masses), dampers etc. Springs and Beams are the elements which don't have the ability to absorb all energy. They rather resist that force applied against them. Dampers are the elements that have the ability to absorb energy. Springs are most commonly characterized according to their stiffness values. The Stiffness, k , is defined as the resistance offered by a spring to deformation. It is not only limited to springs. Every object in this universe possesses some stiffness. Whenever some force is applied to a spring mass system, it vibrates at its natural frequency. It is dependent on the amount of force applied as well as on the resistance of that system against deformation. It is a general trend that if Stiffness value is increased, the natural frequency of the system also increases and if the mass is increased, the natural frequency decreases. This experiment is performed to see the behavior of the system under changing conditions of mass and certain other factors.[1]

Purpose of Experiment:

The purpose of the experiment was to familiarize the students with the concepts of Spring stiffness, natural frequency of cantilever beam and mass moment of inertia of any irregular shaped object. And to see the behavior of spring and cantilever beam under different loading conditions.

Objective of Experiment:

- Calculations of Spring Stiffness using Analytical and experimental methods
- Calculating natural frequency values by using Numerical, Experimental and Analytical methods
- Calculations of the Mass moment of inertia of cylinder using analytical method and experimental method i.e. trifilar pendulum method

PROCEDURE

Analytical Procedure

For Spring:

- The following formula was used to calculate the Stiffness of the spring:

$$k = \frac{Gd^4}{8D^3n_a} \quad \text{Eq1}$$

- **Where:**
 - **G** = shear modulus of spring material.
 - **d** = diameter of the spring wire
 - **D** = Spring mean diameter
 - **n_a** = Number of active coils
 - Units for spring stiffness in SI system in **N/m (Newton/meter)**
- Shear Modulus G can be found using **Young's modulus (E)** of material and the **Poisson's ratio (v)**

$$G = \frac{E}{2(1+v)} \quad \text{Eq2}$$

For Cantilever Beam:

- The following equations was used to calculate the natural frequency of the cantilever beam:

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{Eq3}$$

$$\text{(Deflection of the beam)} \delta = \frac{PL^3}{3EI} \quad \text{Eq4}$$

$$\text{(Stiffness)} k = \frac{P}{\delta} = \frac{3EI}{L^3} \quad \text{Eq5}$$

$$\omega_n = k_i \sqrt{\frac{3EI}{mL^3}} \quad \text{Eq6}$$

Where,

- k_i = Mode Factor
- E = Elastic Modulus
- L = Distance to mass from fixed end
- m = mass at the tip of beam
- I = Area moment of Inertia of beam

For Trifilar Pendulum:

- Following equation can be used to calculate the mass moment of inertia of hollow cylinder

$$J = \frac{m}{2} * (R_{in}^2 + R_{out}^2) \quad \text{Eq7}$$

- m =Mass of the body= 836 g
- R_{in} = Inner radius of cylinder= 2.2 In
- R_{out} =Outer radius of cylinder= 2.375 In

Numerical Procedure

For Cantilever Beam:

- A bar was designed in SolidWorks according to specified dimensions.
- Then, its one end was fixed to make it a Cantilever beam.
- A force was then applied to the tip of the cantilever beam in the program and the system was simulated.
- It gave us the natural frequency of the beam.
- The same procedure was done using different forces at the tip of the beam.
- The numerical and experiment values of the natural frequency of the beam were then compared.

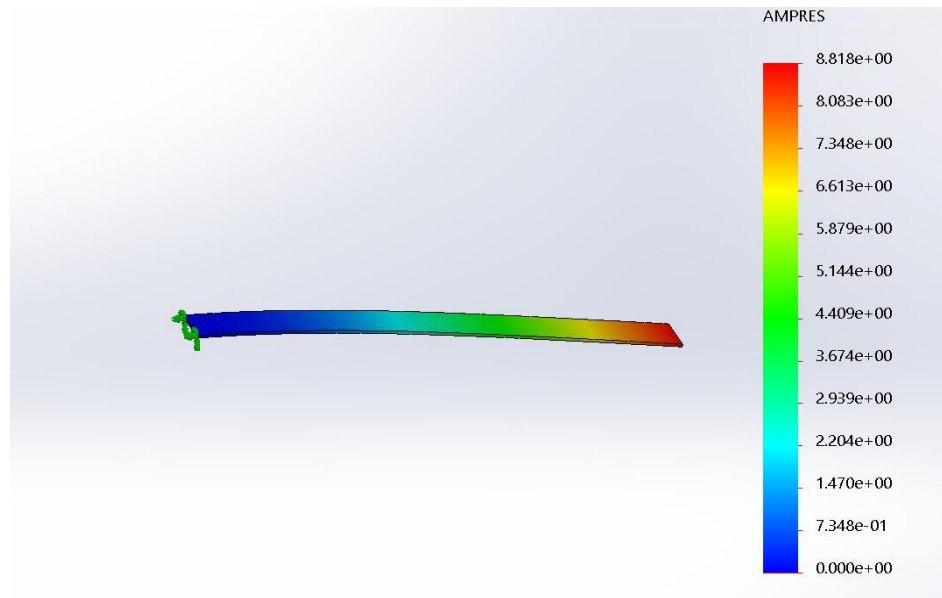


Figure 1 Natural Frequency Pattern of Beam on SolidWorks (1)

This SolidWorks simulation shows the behavior of natural frequency of the Cantilever beam on applying zero mass at the beam. We can observe higher frequency at the free end and zero frequency at its fixed end.

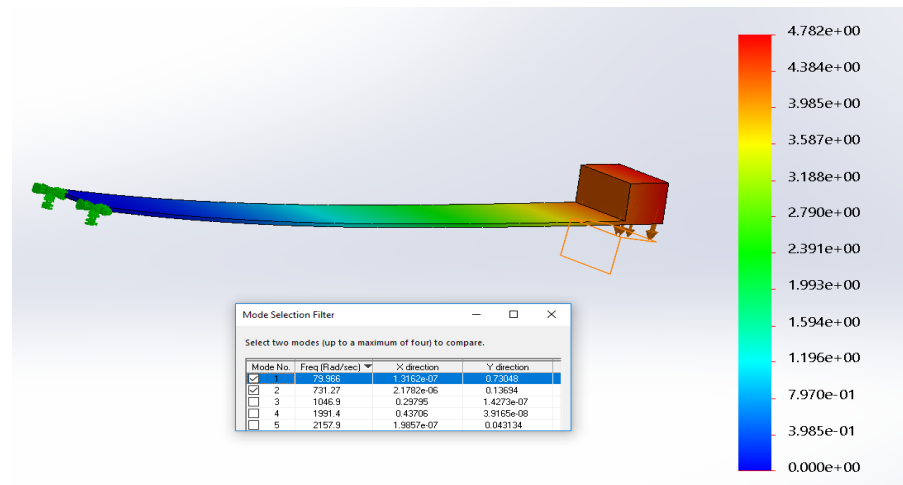


Figure 2 Natural Frequency Pattern of Beam on SolidWorks (2)

This Solid-Works simulation shows that natural frequency behavior of Cantilever beam with applying 100g. From this simulation, we can infer that there will be higher frequency at the free end as compared to the fixed end.

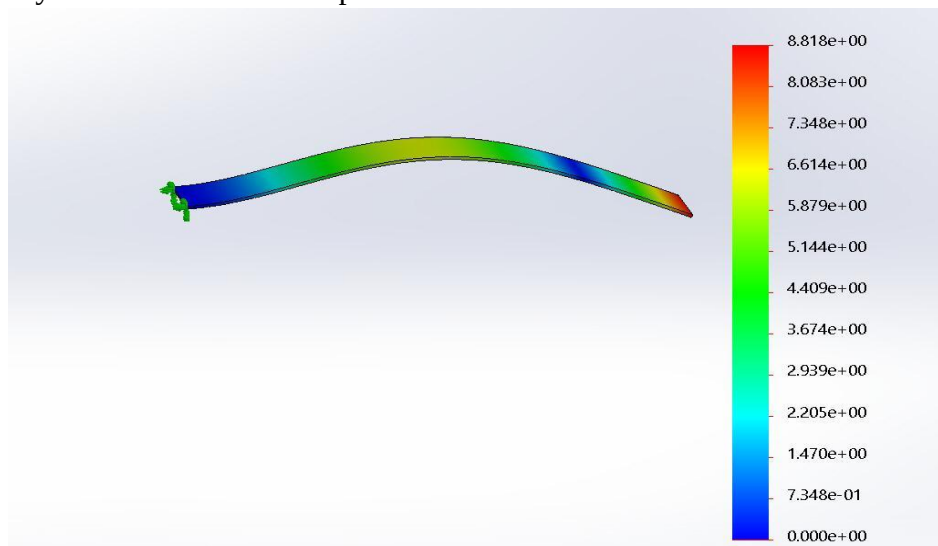


Figure 3 Natural Frequency Pattern of Beam on SolidWorks (3)

This simulation shows the pattern of beam when load is applied at the middle of this Cantilever beam.

Experimental Procedure

For Spring:

- The test spring was taken and its one end was fixed so that it came in vertical position.
- Some weight was applied to the other end of the spring.
- The value of the deflection was noted at this Weight.
- Then, more weights were applied and at each weight, the value of the deflection was noted.
- The graph between Force and Deflection was then plotted.

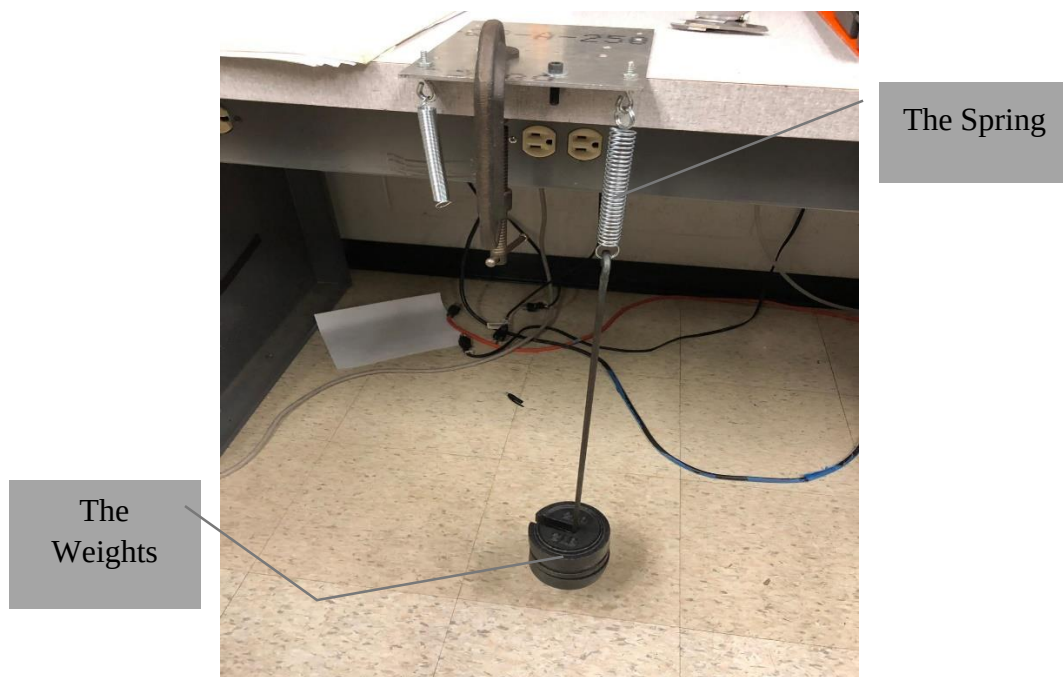


Figure 2 Weights Hanging on Spring

Figure 4 refers to the way how the measurement went, by starting with 2lb, and adding 1 lb in top of it until the weight gets 8 lb. Also, recording the length of the spring while adding any weight.

For Cantilever beam:

- The Cantilever beam was taken and an accelerometer was attached to its tip.
- A force was then applied on the tip of the beam to produce some acceleration. The accelerometer was connected to the computer. The graph for that acceleration was plotted on the computer using MATLAB.
- Using that plot obtained from MATLAB, the natural frequency of the beam was calculated by observing the number of oscillations per second.
- Again, different values of forces were applied on the beam and the whole procedure was repeated.



Figure 3 Cantilever Beam (100g_A)

As showing in Figure 5 that the experiment supported by Matlab recording which gives all actions of the each weight that applied in the top of the beam.

For Trifilar Pendulum:

- Place the required object, whose moment of inertia is to be calculated, on the center of trifilar pendulum apparatus
- Note down the time for 5 Oscillations
- Repeat this process for three times and note down the time for each
- Calculate the moment of inertia of the body by putting values in the formula given below.

The following equation was used to find the mass moment of Inertia:

$$J = \frac{gT^2 r_o^2 (m_o + m)}{4\pi^2 l} - J_o \quad \text{Eq8}$$

Where:

- g = Acceleration due to gravity
- T = Time Period
- r_o = Radius of the disc
- m_o = Mass of the disc
- m = Mass of part being measured
- l = Suspension Wire length
- J_o = Mass moment of inertia of disc



Figure 6- Trifilar Pendulum

As showing in figure 12 that the experiment based on three wires with a height of 26.5 in that connected to a flat disk with diameter of 19.35 in. Also, the cylinder with height of 7 in putting on the top and center of the flat plate.

DATA AND RESULTS

Inputs

For Spring:

In experimental method, applied mass was the input.

In Analytical method, following terms were the inputs.

$G = 77 \text{ GPa}$

$d = 0.06225 \text{ m}$

$D = 0.7240 \text{ m}$

$n_a = 26$

This table contains the Force and deflection data of the spring. It is clear from the table that as applied mass increases, deflection in the spring also increases and their ratio gives us spring stiffness.

Table 1 Spring Stiffness

Mass-m (Kgs)	Deflection-x (meters)	Stiffness-k (Newton/meter)
0.9	0.010922022	807.5427907
1.36	0.018542037	718.7991233
1.81	0.02489205	712.597
2.26	0.03479807	636.4720876
2.72	0.044196088	603.1302989
3.62	0.060452121	586.8445882

For Cantilever Beam:

In both Experimental and analytical methods, applied mass on the tip of beam, mass of beam itself, Area moment of Inertia, Young's modulus of beam and length of the beam were the Inputs.

For Pendulum:

In Analytical method, inner diameter, outer diameter, mass and length of hollow cylinder were known as inputs.

In Experimental method, Time period, radius of disc, mass of disc, mass of cylinder, Suspension wire length and mass moment of inertia of the disc were known

Outputs

In experimental method, Spring deflection curve was the output, the slope of which gives us stiffness of the Spring. This graph shows that there is straight line proportionality between deflection in the spring and the applied load.

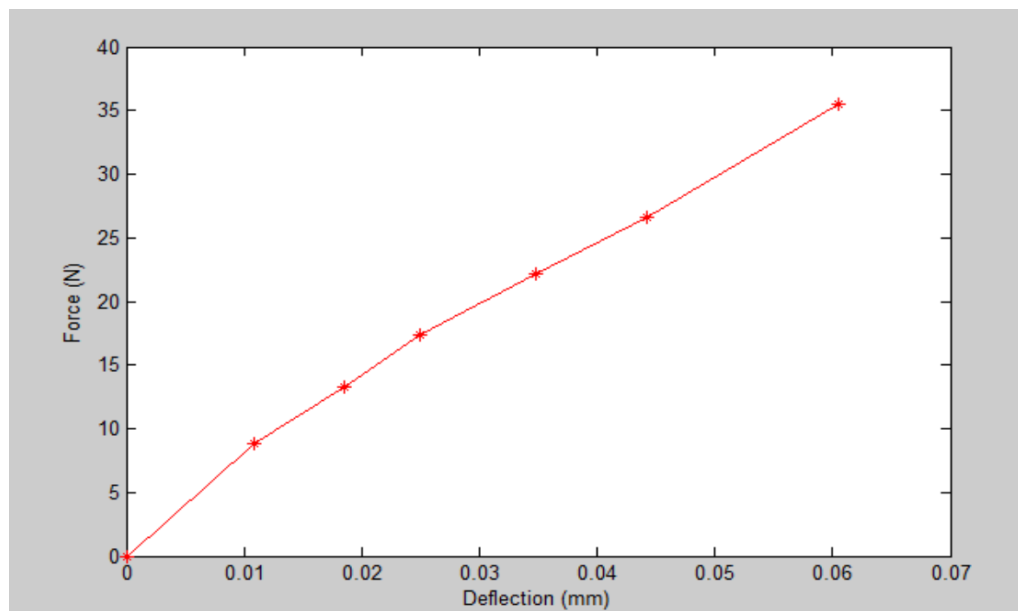


Figure 7 Force Deflection Curve for spring

$$\text{Average of Stiffness} = \sum K_i / n$$

$$\mu = 677.56$$

In Analytical method, Spring stiffness comes out to be $K=14746 \text{ N/m}$

In this experiment, we observed the deflection in the spring as output on applying different masses

For Cantilever Beam:

In both experimental and Analytical methods, Stiffness of the beam material, deflection in the beam and Natural frequency of the beam were calculated as the outputs. This table contains natural frequency values of the Cantilever beam with corresponding applied masses and position of the applied mass.

Table 2 Analytical Data for Cantilever Beam

Mass(grams)	Distance (L-mm)	Stiffness (K)	Natural Frequency
62	203.2	2331.297724	193.9112585
82	203.2	2331.297724	168.6133448
112	203.2	2331.297724	144.2745932
162	203.2	2331.297724	119.961355
162	152.4	5526.039049	184.6925882
162	101.6	18650.38179	339.3019504
162	50.8	149203.0543	959.6908399

Table 3 Experimental Data for Cantilever Beam

Sr.No	Mass(grams)	Distance (mm)	Stiffness (k)	Natural Frequency
1	62	203.2	2331.297724	15
2	82	203.2	2331.297724	12

3	112	203.2	2331.297724	11
4	162	152.4	3688.867065	9
5	162	101.6	7468.974339	12
6	162	50.8	27156.98731	14
7	162	203.2	2331.297724	7

This table shows natural frequency values determined from the accelerometer plots in MATLAB corresponding to different applied masses at different positions.

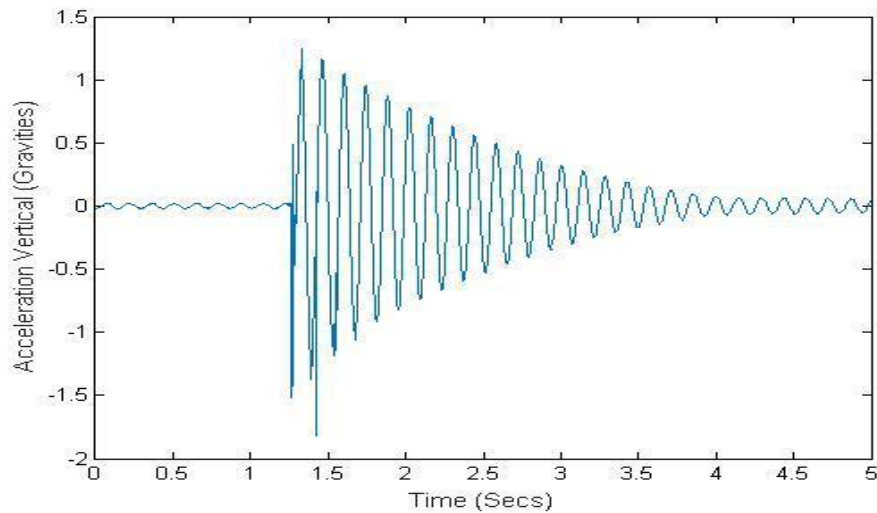


Figure 4 Acceleration Plot of Cantilever Beam (100g_A)

This accelerometer plot indicates that altitude of these vibrations will be lower on applying mass of 100 grams as compared to the lower masses.

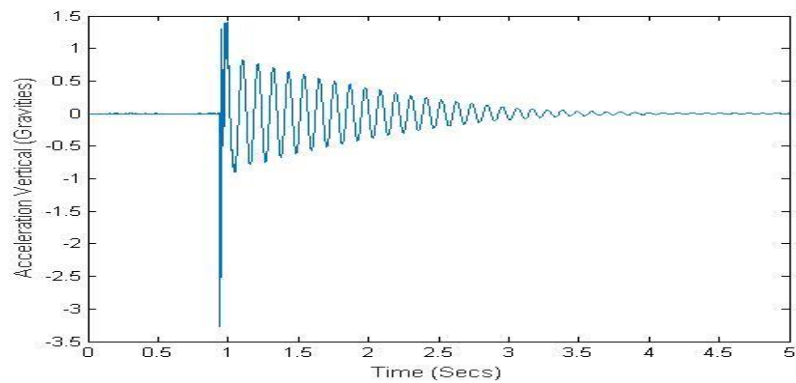


Figure 5 Acceleration Plot of Cantilever Beam (100g_B)

The behavior of the accelerometer graph shows that frequency will be lower when mass of 100 grams is applied at the distance of 6 inches as compared to the mass applied on the free end.

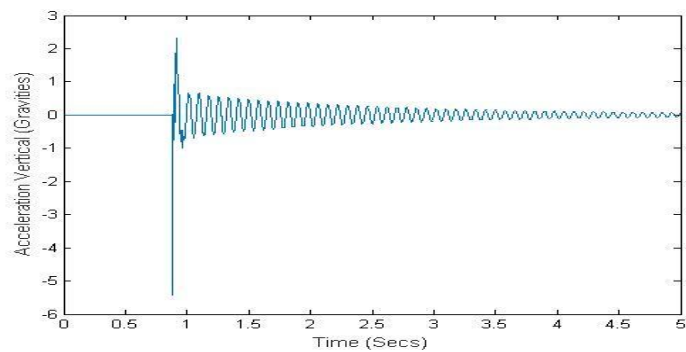


Figure 6 Acceleration Plot of Cantilever Beam (100g_C)

This graph shows that frequency is increasing as we move towards the fixed end of Cantilever beam.

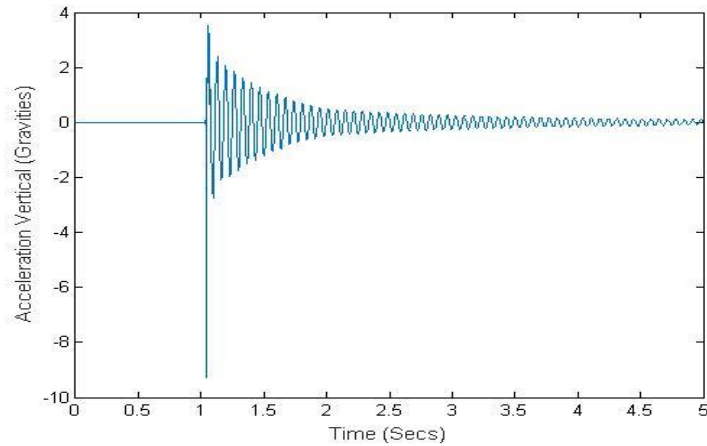


Figure 7 Acceleration Plot of Cantilever Beam (100g_D)

The graph shows that Natural frequency of the beam was high on applying 100 gram mass at 2 inches distance from the fixed end.

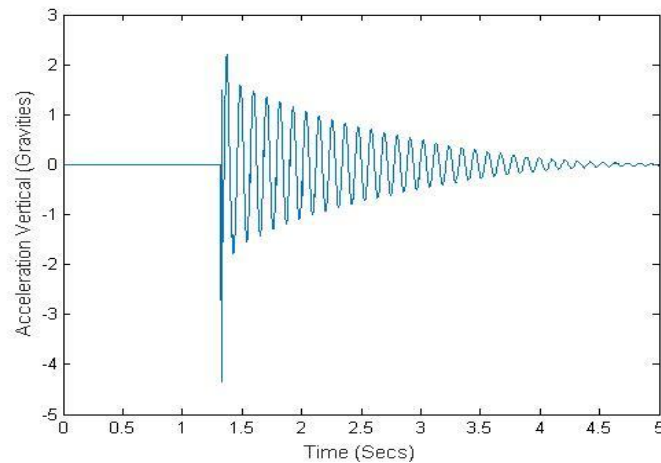


Figure 8 Acceleration Plot of Cantilever Beam (50g_A)

The accelerometer graph indicates that there will be greatest frequency when mass of 50 grams is applied at the free end of the cantilever beam.

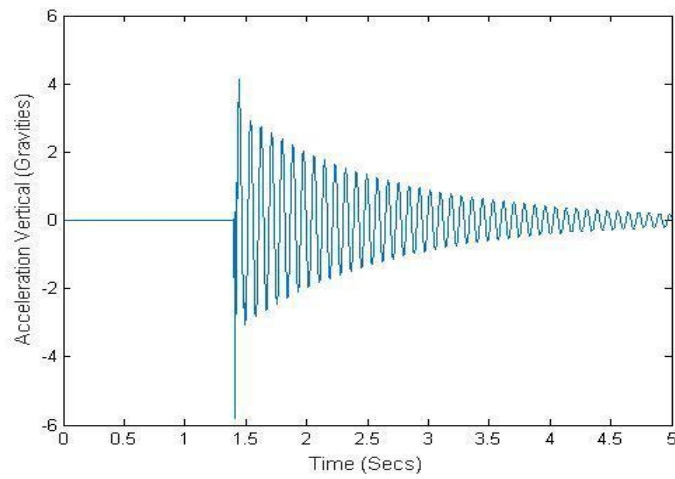


Figure 9 Acceleration Plot of Cantilever Beam (20g_A)

The graph shows that peak of the oscillations will be greater as value of applied mass decreases at the free end of cantilever beam.

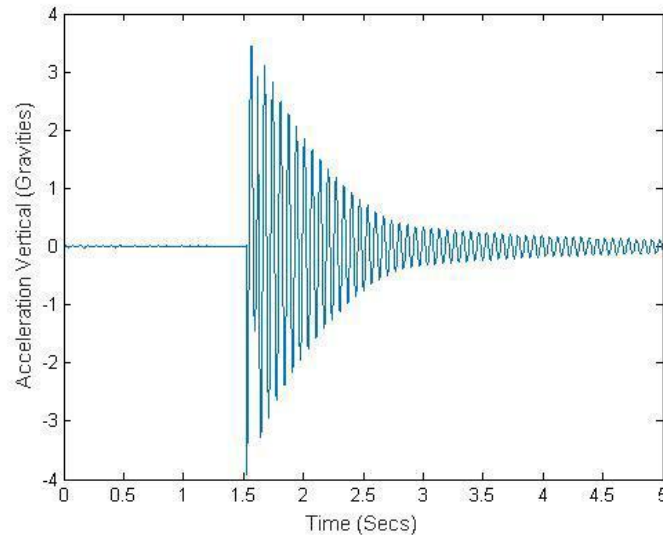


Figure 10 Acceleration Plot of Cantilever Beam (0g_A)

Figure 11 shows that as there is no applied mass on the free end, then altitude of vibrations will be higher and frequency will also be greater.

For Pendulum:

In both experimental and Analytical methods, mass moment of inertia of the cylinder was calculated as the Output. This table shows the mass moment of inertia of cylinder corresponding to different Time Period taken from experimental data.

Table 4 Mass moment of inertia for Cylinder

T (Time Period)	J(Mass moment of Inertia of Cylinder)
1.794	0.011564719
1.856	0.014118843
1.9	0.015984087

From analytical method, mass moment of inertia of Cylinder came out to be:

$$\mathbf{J= 0.00280 \text{ kgm}^2}$$

Error Analysis

- This difference in the Values of Spring stiffness from Experimental and Analytical method was due to human error in noting the values of deflection. This error is also because of inconsistent properties of the Spring material.
- The error in the experimental and analytical values of the Natural frequency was due to environmental conditions and human error in reading the natural frequency values from MATLAB data.
- The error in analytically and experimentally calculated values of moment of inertia of the Cylinder was due to some human error in recording the oscillations time, and also human error in measuring the mass and radius values of the cylinder

Discussion

We performed these three experiments. Firstly, we performed experiment to study the behavior of spring by applying different loads. We compared the values of stiffness of Spring calculated through Analytical and experimental methods.

Secondly, we observed the behavior of Cantilever beam on applying different loads at different positions and checked the natural frequency of beam. We compared the results of natural frequency calculated through Analytical, Numerical and Experimental Methods.

Thirdly, we calculated mass moment of inertia of cylinder by using pendulum method and then compared its value with those calculated through analytical method.

There were some differences in the values calculated from all the methods in all the three experiments due to some reasons mentioned above.

CONCLUSION:

In 1st experiment, we concluded that as we go on increasing the applied weight, deflection in the spring also increases. We observed this behavior by plotting Force deflection curve for the spring, the slope of which gives us Spring stiffness.

In 2nd experiment, we observed that as we apply mass on the tip of the Cantilever beam, some oscillations are generated on this free end. By using these oscillations, we calculate natural frequency of the beam.

In 3rd experiment, we conclude that this trifilar pendulum method was quite efficient to calculate the mass moment of inertia of any irregularly shaped object.

Future work

This spring method can easily be used to find out the spring constant on any elastic material. Numerical method can be used to find out the oscillations of any type of beam by applying any load. Trifilar pendulum method can easily be used to find out the mass moment of inertia of those bodies and objects who have very complex geometry. [5]

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APPENDICES

APPENDIX-A

Cantilever Beam Deflection Formula

The formula of cantilever beam deflection of different types of beam is shown in the figure 14 below.

BEAM DEFLECTION FORMULAE			
BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{\max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{\max} = \frac{Pa^3}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load w (N/m)	$\theta = \frac{wl^3}{6EI}$	$y = \frac{wx^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{\max} = \frac{wl^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity w_0 (N/m)	$\theta = \frac{w_0 l^3}{24EI}$	$y = \frac{w_0 x^2}{120EI}(10l^3 - 10l^2 x + 5lx^2 - x^3)$	$\delta_{\max} = \frac{w_0 l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{\max} = \frac{Ml^2}{2EI}$

Figure 14 Beam Deflection Formula

APPENDIX-B

Beam Deflection and Slope

The formula of the slope and deflection of different type of beam is shown below in the figure 15.

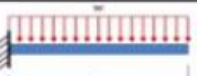
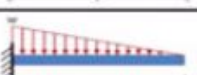
Sr. No.	TYPE OF BEAM	MAX. BM	SLOPE	DEFLECTION
1		M	$\theta = \frac{ML}{EI} = \frac{ML}{EI}$	$\delta = \theta \times \frac{L}{2} = \frac{ML^2}{2EI}$
2		WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\delta = \theta \times \frac{2L}{3} = \frac{WL^3}{3EI}$
3		$\frac{WL^2}{2}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{6EI}$	$\delta = \theta \times \frac{3L}{4} = \frac{WL^4}{8EI}$
4		$\frac{WL^2}{6}$	$\theta = \frac{ML}{4EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{4L}{5} = \frac{WL^4}{30EI}$
5		$\frac{WL}{4}$	$\theta = \frac{ML}{4EI} = \frac{WL^2}{16EI}$	$\delta = \theta \times \frac{L}{3} = \frac{WL^3}{48EI}$
6		$\frac{WL^2}{8}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{5L}{16} = \frac{5WL^4}{384EI}$

Figure 15 Beam Deflection and Slope Formula

APPENDIX-C

Matlab code for the plot of force vs deflection graphs for a spring

```
F = [0 8.82 13.328 17.328 22.148 26.656 35.476];  
deflection_meters = [ 0 0.010922022 0.018542037 0.02489205 0.03479807  
0.044196088 0.060452121];
```

```
plot(deflection_meters,F,'r*-')  
xlabel('Deflection (m)')  
ylabel('Force (Newton)')
```